

IMSc/2001/01/06
hep-ph/0101242

Some Phenomenological Aspects of the ($n + m + 1$) dimensional Brane World Scenario with an m -form Field

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ABSTRACT

In the $D = (n + m + 1)$ dimensional brane world scenario with m compact dimensions, the radion modulus can be stabilised by a massive bulk m -form antisymmetric field. We analyse some of the phenomenological aspects of this scenario. We find that the radion mass is smaller than the TeV scale, but larger than that in the case where the radion modulus is stabilised by a bulk scalar field. From the macroscopic n dimensional spacetime point of view, the m -form field mimics a set of p -form fields. We analyse the mass spectrum of these fields. The lowest mass is $\gtrsim TeV$ whereas, for any bulk or brane field, the excitations in the compact space have Planckian mass and are likely to reintroduce the hierarchy problem. Also, we analyse the couplings of the m -form field to the matter fields living on a brane. The present results are applicable to more general cases also.

PACS numbers: 11.10.Kk, 04.50.+h, 11.25.Mj

1. Randall and Sundrum have recently proposed a simple five dimensional brane world scenario to solve the hierarchy problem [1]. Here, the extra spatial dimension has the topology $\mathbf{S}^1/\mathbf{Z}_2$ in which two branes are located at the fixed points of the \mathbf{Z}_2 with one of them representing our universe. For suitable bulk and brane potentials, the spacetime metric becomes warped which essentially solves the hierarchy problem. See [1] for details.

However, it is crucial to stabilise the size of \mathbf{S}^1 , the radion modulus, at the required value with no fine tuning. Goldberger and Wise have shown [2] that the radion modulus can indeed be stabilised by a massive bulk scalar field. See [2, 3] for details. (For radion stabilisation through quantum effects at non zero temperature, see [4].)

Various phenomenological aspects of this scenario have been analysed. In particular, the radion mass spectrum have been analysed both perturbatively, in the background of the warped metric [5, 6], and exactly, including the back reaction of the scalar field on the warped metric [3, 7, 8]. Fluctuation spectrum of a bulk scalar field [9] and a vector field [10, 11], as well as their couplings to matter fields living on a brane, have also been analysed in the background of the warped metric.

Alternatively, the radion modulus can also be stabilised [12] by a massive bulk m -form antisymmetric field ¹ in $D = (n + m + 1)$ dimensional spacetime with m compact dimensions, with $n = 4$ corresponding to the observable case. Such massive m -form fields appear naturally in the ten dimensional massive type IIA supergravity [14], where $m = 2$, or in the ‘new massive type IIA supergravity’ [15], where $m = 1, 3$. Although it is not clear whether the present scenario can be realised in such fundamental theories, it is nevertheless important to analyse the phenomenological aspects, in particular those that can distinguish the present scenario from that of [1, 2].

In this letter, we analyse these aspects. We analyse the radion mass spectrum in the background of the warped metric ². Note that from the macrosocopic n dimensional spacetime point of view, the m -form field mimics a set of p -form fields. We analyse the mass spectrum of these fields. Also, we analyse the couplings of the m -form field to the matter fields living on a brane.

¹The m -form fields have also been considered in other contexts [13].

² A rigorous analysis, as in [7, 8], requires an exact solution including the back reaction of m -form field on the metric which, however, is not known at present. Hence, we analyse the spectrum perturbatively as in [5, 6].

The main distinguishing features we find are the following. We find that the radion mass is smaller than the TeV scale, but larger than that in [5, 6, 7, 8]. Furthermore, there will be $\frac{m!}{p!(m-p)!}$ number of p -form fields, $1 \leq p \leq \min(n, m)$, in the macroscopic n dimensional spacetime. Their fluctuations form a tower of fields, due to the excitations in the y direction, and due to those in the m -dimensional compact space. Their masses are obtained. They are independent of p and the lowest one is of the order of, but larger than, the TeV scale. We also find that for any bulk or brane field, *e.g.* graviton, m -form field, or Higgs, the excitations in the compact space have Planckian mass and are likely to reintroduce the hierarchy problem.

Throughout in the following, we first present the results in a form applicable to the general case, and then specialise to the particular case under study. They are then applicable to the case of a D dimensional warped spacetime with bulk m -form fields for arbitrary values of D and m and, thus, generalise the analysis of [6, 9, 10, 11].

The plan of the paper is as follows. We present briefly the relevant details of our scenario. We then analyse the radion mass spectrum, fluctuation spectrum of the m -form field, and their couplings to the matter fields living on a brane. We conclude by mentioning a few open issues.

2. We consider $D = (n+m+1)$ dimensional spacetime, $m \geq 1$, containing flat $(n+m-1)$ dimensional branes, with topology $\mathbf{R}^{n-1} \times \mathbf{T}^m$. We assume that \mathbf{T}^m is of D dimensional Planckian size. Thus, on a macroscopic scale, the branes are $(n-1)$ dimensional, with $n = 4$ corresponding to the observable case.

In this letter, we study the Randall-Sundrum configuration [1] with the radion modulus stabilised by the component of the m -form field along \mathbf{T}^m [12].³ Thus, the transverse y direction is a circle, with $-y_1 \leq y \leq y_1$; the points (x^μ, y) and $(x^\mu, -y)$ are identified; and there are two branes located at $y = 0$ and $y = y_1$, which are referred to in the following as Planck and TeV branes respectively. The bulk fields are the metric g_{MN} and a totally antisymmetric m -form field $B_{M_1 \dots M_m}$, of mass Ω . The relevant action is given

³In our notation, $x^M = (x^\mu, y)$ denote the D dimensional spacetime coordinates, x^μ , $\mu = 0, 1, \dots, (D-2)$, the brane worldvolume coordinates, and y the transverse spatial coordinate. The signature of the metric is $(-, +, +, \dots)$. The Riemann tensor is $R_{NKL}^M = \partial_K \Gamma_{NL}^M + \dots$. The D dimensional Planck mass M_{Pl} is set to unity, and the dimensionful quantities, here and in the following, are all taken to be of $\mathcal{O}(1)$ unless mentioned otherwise.

by

$$S = \int d^D x \sqrt{-g} \left(\frac{R}{4} - \frac{G^2}{2(m+1)!} + V_0 - \frac{\Omega^2 \chi}{2m!} - \sum_{I=0,1} \delta(y - y_I) \Lambda_I(\chi) \right), \quad (1)$$

where $g = \det(g_{MN})$, $G = dB$ is the field strength for B , V_0 is a positive constant, and $\chi \equiv B_{M_1 \dots M_m} B^{M_1 \dots M_m}$. Also, y_I are the locations of the branes, and Λ_I the brane potentials.

The background metric, ignoring the back reaction of B , is given by [1]

$$ds^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad (2)$$

where $A = -k|y|$, $k^2 = \frac{4V_0}{(D-1)(D-2)}$, and $\Lambda_0 = -\Lambda_1 = (D-2)k$. The scale on the brane at $y = 0$ is taken to be the D dimensional Planck scale. Then the scale on the brane at $y = y_1$ is given by e^{-ky_1} , which is $\mathcal{O}(TeV)$ if $ky_1 \simeq 40$.

As shown in [12], following the analysis of [2], the radion modulus y_1 can be stabilised at the required value with no fine tuning by the component of the m -form field along $\mathbf{T}^{\mathbf{m}}$. Briefly, the relevant details are as follows.

Let the m -form field B have non vanishing components only along $\mathbf{T}^{\mathbf{m}}$, and the brane potentials be such as to enforce the boundary conditions $\chi = \chi_0$ (χ_1) at $y = 0$ (y_1). Furthermore, let $\frac{\chi_0}{m!}, \frac{\chi_1}{m!} \ll 1$ so that the back reaction on the metric can be consistently neglected. The m -form field equation in the background metric (2) can then be solved. Substituting the solution into the action and integrating over the y coordinate then yields an effective potential U for the modulus y_1 . Defining

$$\gamma = \sqrt{\frac{\chi_1}{\chi_0}}, \quad \frac{K}{k} = \frac{n+m}{2} + \tilde{\nu}, \quad \epsilon = -\frac{n+m}{2} + \tilde{\nu} \quad (3)$$

where $\tilde{\nu} = \sqrt{\frac{(n-m)^2}{4} + \frac{\Omega^2}{k^2}}$, the potential $U(y_1)$ can be written as

$$U(y_1) = \frac{\chi_0(u_1 + u_2)}{m!(1 - e^{-(K+\epsilon k)y_1})} \quad (4)$$

where $u_1 = e^{-(K-\epsilon k)y_1} (K - mk)(e^{-\epsilon k y_1} - \gamma)^2$ and $u_2 = (m + \epsilon)k(1 - \gamma e^{-K y_1})^2$. The radion modulus is stabilised at $y_1 \equiv y_{1s}$ where the potential $U(y_1)$ is minimum. The required value for y_{1s} can then be obtained with no fine tuning (see [12] and below).

3. We now analyse the radion fluctuations and determine its mass. A rigorous analysis of various fluctuations is given in [7, 8] which, however, requires an exact solution that includes the back reaction of the radion stabilising field on the metric. Such solutions are known in the case of the bulk scalar field, but not in the present case. Hence, we analyse the radion fluctuations perturbatively as in [6], neglecting the back reaction on the metric.

The radion field $T(x)$ modifies the metric (2) as follows:

$$ds^2 = e^{-2k|y|T(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + T^2(x) dy^2. \quad (5)$$

One next calculates the Ricci scalar R and reduces the gravity action from D to $(D - 1)$ dimensions. For this purpose, consider a general metric of the form

$$ds^2 = e^{P(x,y)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{Q(x,y)} h_{ab}(y) dy^a dy^b$$

where $\mu, \nu = 0, 1, \dots, (d_1 - 1)$ and $a, b = 1, 2, \dots, d_2$. After some algebra, the Ricci scalar R for the above metric can be written compactly as

$$R = R_{\tilde{g}} - d_2 \nabla_{\tilde{g}}^2 Q - \frac{d_2(d_2 + 1)}{4} e^{-P} (\nabla_g Q)^2 + (d_1, P, g) \longleftrightarrow (d_2, Q, h) \quad (6)$$

where the last part on the right hand side of (6) is obtained from the first by interchanging d_1 and d_2 , P and Q , and $g_{\mu\nu}$ and h_{ab} . $R_{\tilde{g}}$ and $\nabla_{\tilde{g}}^2 Q$ are the Ricci scalar and the Laplacian on Q for the conformally transformed metric $\tilde{g}_{\mu\nu} = e^P g_{\mu\nu}$. Explicitly,

$$\begin{aligned} R_{\tilde{g}} &= e^{-P} (R_g - (d_1 - 1) \nabla_g^2 P - \frac{(d_1 - 1)(d_1 - 2)}{4} (\nabla_g P)^2) \\ \nabla_{\tilde{g}}^2 Q &= e^{-P} (\nabla_g^2 Q + \frac{(d_1 - 2)}{2} (\nabla_g P)(\nabla_g Q)) \end{aligned}$$

where the subscript g denotes that the corresponding quantity is to be calculated using the metric $g_{\mu\nu}$. Similarly for $R_{\tilde{h}}$ and $\nabla_{\tilde{h}}^2 P$.

The Ricci scalar R can now be calculated for the metric (5) where $d_1 = D - 1$, $d_2 = 1$, $P = -2k|y|T(x)$, $Q = 2\ln T(x)$, and $h_{11} = 1$. One then substitutes this expression in the gravity part of the action, and integrates over the y coordinate. Carrying out these steps, which are straightforward, yields the following $(D - 1)$ dimensional action for the radion field T :

$$S = \int d^{D-1}x \sqrt{-g} \left(\frac{(1 - e^{-(D-3)ky_{1s}T})R_g}{2k(D-3)} - \frac{2(D-2)(\nabla_g e^{-\frac{D-3}{2}ky_{1s}T})^2}{k(D-3)^2} \right)$$

where y_{1s} is the stabilised value of the modulus. However, T is not canonically normalised. The canonically normalised radion field, denoted by ϕ , is instead given by

$$\phi = f e^{-\frac{D-3}{2} k y_{1s} T}, \quad f \equiv \sqrt{\frac{4(D-2)}{k(D-3)^2}}. \quad (7)$$

The resulting $(D-1)$ dimensional action for the field ϕ then becomes

$$S = \int d^{D-1}x \sqrt{-g} \left(\frac{(1 - \frac{\phi^2}{f^2}) R_g}{2k(D-3)} - \frac{(\nabla_g \phi)^2}{2} \right). \quad (8)$$

The stabilisation mechanism for the radion modulus induces a potential U for the radion field which has a minimum at $y_1 \equiv y_{1s}$ corresponding to $\langle T \rangle_{VEV} = 1$. The radion mass is then given by

$$m_{radion}^2 = \frac{\partial^2 U}{\partial \phi^2}(y_{1s}). \quad (9)$$

The above expressions are general and are valid for any value of D , irrespective of the stabilisation mechanism. For example, they reduce to those given in [6] for $D = 5$. For the case considered here, $D = (n + m + 1)$ and the radion potential U is given by equation (4) [12]. m_{radion}^2 can then be obtained from (9).

However, obtaining exact analytic expressions for y_{1s} and m_{radion}^2 for the potential U given in (4) is difficult. Hence, we proceed as follows. (The algebraic steps involved in obtaining equations (10)-(12) below are straightforward and, hence, are omitted.) Rearranging the terms suitably, we first rewrite U as

$$U = \frac{(m + \epsilon)k\chi_0}{m!} + \frac{\chi_0 e^{-(K - \epsilon k)y_1}}{m!} \left(\frac{(K + \epsilon k)(e^{-\epsilon k y_1} - \gamma)^2}{(1 - e^{-(K + \epsilon k)y_1})} - (m + \epsilon)k\gamma^2 \right). \quad (10)$$

The analysis of [2, 6, 12] then essentially amounts to neglecting the last term above. However, a much better approximation is to neglect instead the exponential factor in the denominator. It is then straightforward to obtain y_{1s} and m_{radion}^2 . Neglecting this factor and setting $\frac{dU}{dy_1}$ to zero then gives

$$(K + \epsilon k)^2 e^{-2\epsilon k y_1} - 2\gamma K(K + \epsilon k) e^{-\epsilon k y_1} + \gamma^2 (K - \epsilon k)(K - mk) = 0$$

which is just a quadratic equation for $e^{-\epsilon ky_1}$. The solution $y_1 \equiv y_{1s}$ where U is minimum is then given by

$$ky_{1s} = -\frac{1}{\epsilon} \ln \left(\frac{\gamma(n+m+\epsilon+c)}{n+m+2\epsilon} \right), \quad c = \sqrt{\epsilon^2 + (n+m)(m+\epsilon)} \quad (11)$$

where we have used $K = (n+m+\epsilon)k$ which follows from equation (3). From (11), it can be seen that $ky_{1s} \simeq 40$ can be easily achieved without any fine tuning by choosing ϵ to be small [2, 12]. In the following we assume that ϵ is small and that the parameters γ and ϵ are chosen such that $ky_{1s} \simeq 40$.

One can now evaluate the second derivative of U at y_{1s} and, thereby, obtain the radion mass. Using equations (9), (10), and (11), it is given by

$$m_{radion}^2 = \frac{2(n+m+\epsilon+c)}{(n+m-1)} \left(\frac{\chi_1 \epsilon c}{m!} \right) k^2 e^{-2ky_{1s}} \quad (12)$$

which gives the radion mass in the present scenario where the radion is stabilised by a bulk m -form field.

Consider equation (12). The exponential factor brings the radion mass down to the TeV scale, namely to the scale $ke^{-ky_{1s}}$ on the TeV brane. It is further suppressed by $\frac{\chi_1}{m!}$, which is taken to be $\ll 1$ so that the above analysis is valid where the back reaction is neglected. Hence, the radion mass will be smaller than TeV. These suppression factors are also present in the scenario where radion is stabilised by the bulk scalar field [5, 6].

Consider now the factor (ϵc) . This suppression of the radion mass by the ϵ factor is analogous to that found in the case of the bulk scalar field [5, 6]. However, the exponent of ϵ in these cases are different.

The results for the bulk scalar field case can be obtained from the above expressions by setting $m = 0$ formally. Then, with $m = 0$ and ϵ small, $c \simeq \sqrt{\epsilon}$ and, as found in [5] and mentioned in the footnote 2 of [6],

$$m_{radion}^2 \propto \epsilon c \simeq \epsilon^{\frac{3}{2}}. \quad (13)$$

The above expressions are also valid when the last term in (10) is neglected if the $(m+\epsilon)$ factor in the expression for c is formally set to zero. Then, $c = \epsilon$ for any value of m , and we obtain the result given in [6]:

$$ky_{1s} = -\frac{1}{\epsilon} \ln \gamma, \quad \text{and} \quad m_{radion}^2 \propto \epsilon^2. \quad (14)$$

Note that the exact analysis of [7, 8] also gives the above relation for the radion mass, but the origin of the different scalings in (13) and (14) is not understood [8]. For the case of interest here, $m \neq 0$ and $c \simeq \sqrt{m(n+m)} = \mathcal{O}(1)$. Then

$$m_{radion}^2 \propto \epsilon. \quad (15)$$

Thus, in the perturbative analysis of the present scenario, the radion mass is given by equation (12). It is smaller than TeV but larger, by a factor of $\epsilon^{-\frac{1}{4}}$ ($\epsilon^{-\frac{1}{2}}$), than that obtained in [5, 6] ([6, 7, 8]) where the radion is stabilised by a bulk scalar field.

4. We now consider the fluctuations of the other components of the m -form field. If the m -form field has non vanishing components along p of the \mathbf{R}^n directions, $1 \leq p \leq \min(n, m)$, then, from the macroscopic n dimensional point of view, it mimics a p -form field. The number of such p -form fields is equal to $\frac{m!}{p!(m-p)!}$. Here, we analyse the fluctuation spectrum of these fields.

Consider the general case of an m -form bulk field $B_{M_1 \dots M_m}$, of mass M , in the D dimensional spacetime with the background warped metric given by (2). The action for the m -form field B is given by

$$S = - \int d^{D-1}x dy \sqrt{-g} \left(\frac{G^2}{2(m+1)!} + \frac{\Omega^2 \chi}{2m!} \right) \quad (16)$$

where G and χ are as defined in equation (1). The equation of motion is given by

$$\nabla_M G^{M M_1 \dots M_m} = M^2 B^{M_1 \dots M_m}. \quad (17)$$

For the background metric (2), the action (16) can be written explicitly as

$$S = - \int d^{D-1}x dy e^{(D-1-2m)A} \times \left(\frac{e^{-2A} (G_{\mu\mu_1 \dots \mu_m})^2}{2(m+1)!} + \frac{(\partial_y B_{\mu_1 \dots \mu_m})^2 + \Omega^2 (B_{\mu_1 \dots \mu_m})^2}{2m!} \right), \quad (18)$$

where the μ indices are now contracted using $\eta^{\mu\nu}$. Consider the ansatz

$$B_{\mu_1 \dots \mu_m}(x^\mu, y) = \sum_i B_{\mu_1 \dots \mu_m}^{(i)}(x^\mu) \frac{f_{(i)}(y)}{\sqrt{y_1}} \quad (19)$$

where the functions $f_{(i)}(y)$ satisfy the differential equation

$$- \frac{d}{dy} (e^{(D-1-2m)A} \frac{d}{dy} f_{(i)}) + M^2 e^{(D-1-2m)A} f_{(i)} = m_{(i)}^2 e^{(D-3-2m)A} f_{(i)} \quad (20)$$

and are normalised as follows:

$$\int_{-y_1}^{y_1} \frac{dy}{y_1} e^{(D-3-2m)A} f_{(i)}(y) f_{(j)}(y) = \delta_{ij} . \quad (21)$$

Since the points (x^μ, y) and $(x^\mu, -y)$ are identified, it follows that $f_{(i)}(y)$ must satisfy the conditions $f_{(i)}(-y) = f_{(i)}(y)$, which determines $f_{(i)}(-y)$, and

$$\frac{df_{(i)}}{dy}(0) = \frac{df_{(i)}}{dy}(y_1) = 0 . \quad (22)$$

Therefore, it suffices to obtain $f_{(i)}(y)$ for $0 \leq y \leq y_1$ satisfying the boundary condition (22).

Using equations (19), (20), and (21), the $(D-1)$ dimensional action for the fields $B^{(i)}(x^\mu)$ can now be determined. Partial integrating the $(\partial_y B)^2$ term in (18), and then performing the y integration using the above equations, the $(D-1)$ dimensional action for the fields $B^{(i)}$ becomes

$$S = - \int d^{D-1}x \sum_i \left(\frac{(G^{(i)})^2}{2(m+1)!} + \frac{m_{(i)}^2 (B^{(i)})^2}{2m!} \right) \quad (23)$$

which shows that $m_{(i)}$ is the mass of the $(D-1)$ dimensional m -form fields $B^{(i)}$. (Substituting the ansatz (19) directly into the equation of motion (17) also leads to the equation (20) for $f_{(i)}(y)$, and to the equation of motion for $B^{(i)}$ obtained from (23).)

Consider equation (20) for $y \geq 0$. Using $A = -ky$, its solutions are given in terms of Bessel functions J and Y by

$$f_{(i)} = \frac{e^{\frac{D-1-2m}{2}ky}}{N_{(i)}} \left(J_\nu\left(\frac{m_{(i)}}{k}e^{ky}\right) + b_{(i)} Y_\nu\left(\frac{m_{(i)}}{k}e^{ky}\right) \right) , \quad (24)$$

where $b_{(i)}$ and $N_{(i)}$ are constants and $\nu = \sqrt{\frac{(D-1-2m)^2}{4} + \frac{M^2}{k^2}}$. It follows from equation (21) that the normalisation constant $N_{(i)}$ is given by

$$N_{(i)}^2 = \int_{-y_1}^{y_1} \frac{dy}{y_1} e^{2ky} \left(J_\nu\left(\frac{m_{(i)}}{k}e^{ky}\right) + b_{(i)} Y_\nu\left(\frac{m_{(i)}}{k}e^{ky}\right) \right)^2 . \quad (25)$$

Using (24), the boundary condition (22) implies that

$$b_{(i)} = b\left(\frac{m_{(i)}}{k}\right) = b\left(\frac{m_{(i)}}{k}e^{ky_1}\right) \quad (26)$$

where we have defined $b(x) \equiv -\frac{(D-1-2m)J_\nu(x)+2xJ'_\nu(x)}{(D-1-2m)Y_\nu(x)+2xY'_\nu(x)}$, with the primes denoting the derivatives with respect to x . These relations determine the constant $b_{(i)}$ and the eigenvalues $m_{(i)}$.

The eigenvalues of the masses $m_{(i)}$ can be estimated easily from the above equations. Note that when the radion modulus is stabilised, as in the present case, $ky_1 = ky_{1s} \simeq 40$. The exponent e^{ky_1} is therefore extremely large. It then follows that $m_{(i)}$ and $b_{(i)}$ are extremely small, and that the mass eigenvalues are given approximately by

$$m_{(i)} \simeq x_{\nu i} (ke^{-ky_{1s}}) \simeq x_{\nu i} \times \text{TeV} \quad (27)$$

where $x_{\nu i}$ is the i^{th} zero of the Bessel function J_ν . Hence, it follows that, in the background of the warped metric (2), the fluctuations of the m -form field has a mass spectrum given by (27). The normalisation constant $N_{(i)}$ can also be estimated from (25). Since $b_{(i)}$ is small and $ky_{1s} \gg 1$, $N_{(i)}$ is given approximately by [9, 11]

$$N_{(i)}^2 \simeq \int \frac{dy}{y_1} e^{2ky} J_\nu^2\left(\frac{m_{(i)}}{k} e^{ky}\right) \simeq \frac{e^{2ky_{1s}}}{ky_{1s}}. \quad (28)$$

The above expressions are general and are valid for any value of D , m , and M . For example, they reduce to those given in [9] ([10, 11]) for $D = 5$ and $m = 0$ (1), which corresponds to a bulk scalar (vector) field.

In the present case $D = (n + m + 1)$ and $M^2 = \Omega^2$. Hence, as follows from equation (3), the index $\nu = \tilde{\nu} > \frac{n+m}{2}$. Since the first zero $x_{\nu 1} > \nu$ [16], the lowest mass $m_{(1)} > \frac{n+m}{2} (ke^{-ky_{1s}}) \gtrsim TeV$.

In the present case, the brane is $(n + m - 1)$ dimensional, with topology $\mathbf{R}^{n-1} \times \mathbf{T}^m$, where \mathbf{T}^m is of Planckian size. Hence, from the macroscopic n dimensional spacetime point of view, the Kaluza-Klein (KK) excitations of both the bulk and the brane fields along \mathbf{T}^m will form a tower of fields. Consider the bulk B field. (Other fields such as graviton, Higgs, etc. can also be analysed similarly, and the results are qualitatively the same.) Let \tilde{x} be the coordinates of the n dimensional spacetime, ξ^a and L_a be, respectively, the coordinate and the size of the a^{th} direction of \mathbf{T}^m , and n_a the corresponding KK excitation number. The KK excitations can then be written schematically as

$$B^{(i)}(x^\mu) = \sum_{(n_a)} B^{(n_a, i)}(\tilde{x}) e^{i \sum_a \frac{n_a \xi^a}{L_a}}. \quad (29)$$

Substituting (29) and (19) in the action (16) and integrating over the ξ^a and y coordinates, in any order, one obtains the n dimensional action for the tower of fields $B^{(n_a,i)}$, given by

$$S = - \int d^n \tilde{x} \sum_{n_a,i} \left(\frac{(G^{(n_a,i)})^2}{2(m+1)!} + \frac{m_{(n_a,i)}^2 (B^{(n_a,i)})^2}{2m!} \right) \quad (30)$$

where the masses $m_{(n_a,i)}$ of the fields $B^{(n_a,i)}$ are given by

$$m_{(n_a,i)}^2 = m_{(i)}^2 + M_{KK}^2, \quad M_{KK}^2 \equiv \sum_a \frac{n_a^2}{L_a^2}. \quad (31)$$

(Substituting the ansatz (29) and (19) directly into the equation of motion (17) also leads to the above result.) Note that m_i is of order TeV , as given by (27), whereas M_{KK} is of order M_{pl} if n_a are not all zero.

The above formula applies to all the components of the m -form field which, from the macroscopic n dimensional spacetime point of view, mimics the $\frac{m!}{p!(m-p)!}$ number of p -form fields, $1 \leq p \leq \min(n, m)$. Thus, it follows that their fluctuations form a tower of fields, with masses independent of p and given by (27) and (31). The zero modes along \mathbf{T}^m have masses given by (27) and are of the order of, but larger than, TeV . The KK excitations have masses given by (31) and are of order M_{pl} .

The fact that KK excitations along \mathbf{T}^m have Planckian mass can also be seen as follows. Because of the warped metric (2), the size of \mathbf{T}^m at y is e^{-ky} times the Planck length. Hence, the KK excitations will have a mass of order $e^{ky} M_{pl}^4$. However, their kinetic terms are not in the canonical form and, therefore, the fields need to be rescaled. Upon this rescaling, the masses are suppressed by a factor of e^{-ky} [1]. The KK excitation mass will then be of order M_{pl} . This can be seen explicitly if one substitutes (29) in (16) and integrates over the ξ^a coordinates first.

This phenomenon is generic for any bulk or brane field if the braneworld has compact directions of Planckian size. An important consequence is that it is very likely to reintroduce the hierarchy problem. For example, both graviton and the Higgs field will have KK excitations, with mass of order M_{pl} . From the macroscopic n dimensional spacetime point of view, the zero mode of the Higgs field can couple directly to its own KK excitations through

⁴We thank the referee for the above comments in this paragraph.

the Higgs potential, and also to the graviton KK excitations through their mutual gravitational couplings. Even if the corresponding coupling strengths are suppressed by Planck scale, the loop corrections are still likely to push up the Higgs mass to M_{pl} because of the infinite number of fields present in the KK tower, thereby reintroducing the hierarchy problem. Explicit calculations are currently in progress.

To avoid this problem, one may wish to take the size of $\mathbf{T}^{\mathbf{m}}$ to be $\gtrsim k^{-1}e^{ky_{1s}} \simeq (TeV)^{-1}$, so that it is always larger than the Planck length for $-y_{1s} \leq y \leq y_{1s}$. KK excitation mass would then be of order $ke^{-ky_{1s}} \simeq TeV$, and the lowest excitation, *e.g.* of graviton, may then provide another signature of the model. However, stabilising the size of $\mathbf{T}^{\mathbf{m}}$ at such a value introduces a different hierarchy problem since the fundamental scale is assumed to be M_{pl} ⁵. Moreover, the n dimensional Newton's constant G_N , given by $G_N^{2-n} \propto Vol(\mathbf{T}^{\mathbf{m}})k^{-1}M_{pl}^{n+m-1}$, will not be $\mathcal{O}(M_{pl}^{-1})$. For these reasons, we assume that $\mathbf{T}^{\mathbf{m}}$ is of Planckian size.

5. Consider the couplings to matter fields on the brane. The radion couplings in the case of bulk scalar field have been analysed in detail in [5, 6, 7, 8]. The same analysis applies for the radion couplings in the present case also. Hence, we will not consider it further.

We analyse here the couplings of the other components of the m -form fields to the matter fields living on the brane located at $y = y_*$. Let $\mathcal{O}^{M_1 \dots M_m}$, $M_1, M_2, \dots = 0, 1, \dots, (D-2)$ be the matter field operator on the brane at y_* to which the m -form field couples⁶. The relevant action is given by

$$S_{\mathcal{O}} = \int d^{D-1}x dy \sqrt{-g} (\lambda_D \mathcal{O}^{M_1 \dots M_m}) B_{M_1 \dots M_m} \delta(y - y_*) , \quad (32)$$

where λ_D is the coupling constant.

As will be seen below, this action leads to the $(D-1)$ dimensional action of the form $S_{\mathcal{O}} \sim \int d^{D-1}x (\lambda \mathcal{O}^{\mu_1 \dots \mu_m}) \sum_i B_{\mu_1 \dots \mu_m}^{(i)}$ where the indices are contracted using $\eta^{\mu\nu}$. With no loss of generality, we define the coupling constant λ to be dimensionless, which can always be done by multiplying the operator \mathcal{O} with suitable powers of the Planck mass M_{pl} , which has been set to unity in our notation here. The mass dimensions, denoted by $[]$, of the various quantities can now be obtained: $[\lambda] = 0$ by assumption, and $[B^{(i)}] = \frac{D-3}{2}$

⁵We thank the referee for the above comments in this paragraph.

⁶For example, for fermionic coupling, $\mathcal{O}^{M_1 \dots M_m} \sim \bar{\Psi} \Gamma^{M_1} \dots \Gamma^{M_m} \Psi$ where Ψ denotes the fermion living on the brane and Γ the Dirac matrix.

as follows from (23). Therefore, $[\mathcal{O}] = \frac{D+1}{2}$.⁷ Also, as can be seen from equation (19), $[B] = \frac{D-2}{2}$ and, hence, $[\lambda_D] = -\frac{1}{2}$.

Consider now the action (32). Writing $\mathcal{O}^{M_1 \dots M_m} = E_{a_1}^{M_1} \dots E_{a_m}^{M_m} \mathcal{O}^{a_1 \dots a_m}$, where $E_a^M \propto e^{ky_*}$ are the D -beins, and using equations (19) and (24), the action (32) can be written as

$$S_{\mathcal{O}} = \int d^{D-1}x e^{-\frac{D-1}{2}ky_*} (\lambda \mathcal{O}^{\mu_1 \dots \mu_m}) \\ \times \sum_i \frac{B_{\mu_1 \dots \mu_m}^{(i)}}{N_{(i)}} \left(J_\nu \left(\frac{m_{(i)}}{k} e^{ky_*} \right) + b_{(i)} Y_\nu \left(\frac{m_{(i)}}{k} e^{ky_*} \right) \right)$$

where $\lambda \equiv \frac{\lambda_D}{\sqrt{y_{1s}}}$. Because of the warped nature of the D dimensional space-time, the scale on the brane at y_* is set by $e^{-ky_*} M_{pl}$. Since $[\mathcal{O}] = \frac{D+1}{2}$, the appropriate operator \mathcal{O}_* , with all its mass scales corresponding to the scale e^{-ky_*} on the brane at y_* , is given by

$$\mathcal{O}_*^{\mu_1 \dots \mu_m} = e^{-\frac{D+1}{2}ky_*} \mathcal{O}^{\mu_1 \dots \mu_m}.$$

Using the fact that $b_{(i)}$ is negligible and using (28) for $N_{(i)}$, we then obtain

$$S_{\mathcal{O}_*} \simeq \int d^{D-1}x e^{-k(y_{1s}-y_*)} (\lambda \sqrt{ky_{1s}}) \mathcal{O}_*^{\mu_1 \dots \mu_m} \sum_i B_{\mu_1 \dots \mu_m}^{(i)} J_\nu \left(\frac{m_{(i)}}{k} e^{ky_*} \right) \quad (33)$$

which describes the coupling of the $(D-1)$ dimensional m -form fields $B^{(i)}$ to the matter fields living on the brane at y_* .

From equation (33), the following features can be seen easily: **(i)** The dimensionless coupling constant λ is enhanced by a factor $\sqrt{ky_{1s}}$, which arises from the normalisation constant $N_{(i)}$. **(ii)** Near the Planck brane, $y_* \simeq 0$ and the coupling is suppressed by $e^{-ky_{1s}}$, which also arises from $N_{(i)}$. It is further suppressed by the factor $J_\nu \simeq \left(\frac{m_{(i)}}{k} \right)^\nu \simeq e^{-\nu ky_{1s}}$. **(iii)** Near the TeV brane, $y_* \simeq y_{1s}$ and, hence, both the suppression factors described in **(ii)** are absent. Only the enhancement factor $\sqrt{ky_{1s}}$ is present.

The above expressions are general and are valid for any value of D and m . It is easy to check that they reduce to those given in [10, 11] for $D = 5$ and $m = 1$, which corresponds to a bulk vector field. In the present case,

⁷Thus, for example, since $[\Psi] = \frac{D-2}{2}$ the fermionic operator is given by $\mathcal{O}^{M_1 \dots M_m} = M_{pl}^{\frac{5-D}{2}} \bar{\Psi} \Gamma^{M_1} \dots \Gamma^{M_m} \Psi$, where the Planck mass dependence is chosen so that $[\mathcal{O}] = \frac{D+1}{2}$.

$D = (n + m + 1)$ and the action (33) describes the couplings of the m -form field to the matter fields living on the brane located at y_* .

6. We now conclude by mentioning a few open issues. The radion mass is obtained by a perturbative analysis. A rigorous analysis requires an exact solution including the back reaction of the m -form field on the metric which, however, is not known at present. It is thus important to find such solutions, which may be of interest in their own right. Also, it will be of interest to study other phenomenological features which can distinguish the present scenario from that of [1, 2].

The KK excitations of Planckian mass are generically present if the braneworld has compact directions of Planckian size. They are very likely to reintroduce the hierarchy problem. Explicit calculations of this phenomenon within the present scenario are currently in progress.

However, since Planckian lengths are involved, a consistent study and possible resolution, if any, of this problem requires the knowledge of the underlying theory. It is, therefore, important to answer the question of whether the present scenario can be realised in a fundamental theory, *e.g.* supergravity, string, or M theory. Although the answer is presently not known, it is encouraging that massive m -form fields appear naturally in massive type IIA supergravity theories [14, 15], which are believed to be related to M theory. It is then worthwhile to study whether the present scenario can be realised by some suitable compactification of these theories.

Acknowledgement: It is a pleasure to thank R. K. Kaul for discussions, and also the referee for his/her important and helpful comments.

References

- [1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370, hep-ph/9905221.
- [2] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83** (1999) 4922, hep-ph/9907447.
- [3] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62** (2000) 046008, hep-th/9909134.

- [4] I. Brevik, K. A. Milton, S. Nojiri, and S. D. Odintsov, hep-th/0010205, to appear in Nucl. Phys. **B**.
- [5] C. Csaki, M. Graesser, L. Randall, and J. Terning, Phys. Rev. **D 62** (2000) 045015, hep-ph/9911406.
- [6] W. D. Goldberger and M. B. Wise, Phys. Lett. **B 475** (2000) 275, hep-ph/9911457.
- [7] T. Tanaka and X. Montes, Nucl. Phys. **B 582** (2000) 259, hep-th/0001092. See also, J. Garriga and T. Tanaka, Phys. Rev. Lett. **84** (2000) 2778, hep-th/9911055.
- [8] C. Csaki, M. L. Graesser, G. D. Kribs, Phys. Rev. **D 63** (2001) 065002, hep-th/0008151.
- [9] W. D. Goldberger and M. B. Wise, Phys. Rev. **D 60** (1999) 107505, hep-ph/9907218.
- [10] H. Davoudiasl, J. L. Hewett, and T. G. Rizzo, Phys. Lett. **B 473** (2000) 43, hep-ph/9911262.
- [11] A. Pomarol, Phys. Lett. **B 486** (2000) 153, hep-ph/9911294.
- [12] S. Kalyana Rama, Phys. Lett. **B 495** (2000) 176, hep-th/0010121.
- [13] T. Gherghetta, E. Roessl, and M. Shaposhnikov, Phys. Lett. **B 491** (2000) 353, hep-th/0006251; S. Hayakawa and K. -I. Izawa, Phys. Lett. **B 493** (2000) 380, hep-th/0008111.
- [14] L. J. Romans, Phys. Lett. **B 169** (1986) 374.
- [15] P. S. Howe, N. D. Lambert, and P. C. West, Phys. Lett. **B 416** (1998) 303, hep-th/9707139.
- [16] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, Second Edition, 1944.